

WE
DEMOCRATIZE
ANALYTICS



FRONTLINE
solvers

EXCEL USERS
WEB USERS
DEVELOPERS

*Data Visualization * Data Mining * Simulation / Risk Analysis * Decision Trees * Conventional / Stochastic Optimization*

Optimization

Using Analytic Solver Platform

REVIEW BASED ON
MANAGEMENT SCIENCE

FrontlineSolvers



What We'll Cover Today



- Introduction
 - Session II beta training program goals
- Classic Models for Common Business Situations
- Focus on Modeling with Linear and Integer Constraints
- 'Linearizing' Ratio, Either-Or, If-Then Constraints

Session II Online Beta Training Goals



To familiarize you with the following concepts:

- Art of identifying and formulating constraints
- Designing linear constraints, using binary variables
- How ASP can linearize constraints automatically

To empower you to achieve success

- State of the art tools
- Online educational training
- User guides and video demos

Typical Optimization Applications



Industry

Energy
Chemical
Manufacturing
Transportation
Finance
Agriculture
Health
Mining
Defense
Forestry

Functional Area

Staff planning
Scheduling
Routing
Blending
Capacity planning
Media planning
Supply chain
 Inventory optimization
 Vendor selection
Portfolio optimization
Product mix

Why Focus on Linear Models



- Most common business situations can be modeled with linear functions, sometimes using integer variables.
- A linear program, with a linear objective function and all linear constraints, can be solved quickly to a globally optimal solution.
- Integer variables make a problem non-convex and more difficult to solve, but their structure can be exploited by modern Solvers.
- If there is a choice, use a linear formulation rather than a nonlinear formulation.

Common Model Structures in Business Problems



- Allocation Models – Maximize objective (profit) subject to \leq constraints on capacity.
- Covering Models – Minimize objective (cost) subject to \geq constraints on coverage.
- Blending Models – Mix inputs with different properties to satisfy quality constraints and minimize cost.
- Network Models – For situations with ‘flows’ from sources to destinations.
 - Transportation – Moving goods.
 - Assignment – Matching candidates to positions.
 - Network Models for Process Technologies.

Formulation



- 1) Determine decision variables
 - “What factors are under our control?”
- 2) Determine objective function
 - “What measure are we trying to optimize?”
- 3) Determine constraints
 - “What restrictions limit our choice of decision variables?”
 - Available resources
 - Physical constraints
 - Policy constraints

Allocation Models



- Maximize objective (profit) subject to \leq constraints on capacity.
- Veerman Furniture Company – Page 221 – Powell & Baker.
- Determine the mix of chairs, desks, & tables to maximize profit.



Department	Hours per Unit			Hours Available
	Chairs	Desks	Tables	
Fabrication	4	6	2	1,850
Assembly	3	5	7	2,400
Shipping	3	2	4	1,500
Demand Potential	360	300	100	
Profit	\$15	\$24	\$18	



Allocation Models

- 1) What factors are under our control?
 - The product mix – number of Chairs (C), Desks (D), Tables (T).
- 2) What measure are we trying to optimize?
 - Maximize profit contribution.
- 3) What restrictions limit our choice of decision variables?
 - Production capacity
 - Fabrication
 - Assembly
 - Shipping
 - Demand potential
 - Physical Constraint



Allocation Models

- Decision variables – number of Chairs (C), Desks (D), Tables (T)
- Objective Function – Profit maximization
- Constraints –
 - Fabrication
 - Assembly
 - Shipping
 - Demand potential
 - Non-negativity

Subject to

$$\text{Max } 15C + 24D + 18T$$

$$4C + 6D + 2D \leq 1850$$

$$3C + 5D + 7T \leq 2400$$

$$3C + 2D + 4T \leq 1500$$

$$C \leq 360$$

$$D \leq 300$$

$$T \leq 100$$

$$C \geq 0$$

$$D \geq 0$$

$$T \geq 0$$

Limited Resources

Limited Demand

Physical Constraints



Covering Models

- Minimize objective (cost) subject to \geq constraints on coverage.
- Dahlby Outfitters – Page 226 – Powell & Baker.
- The ingredients contain certain amounts of vitamins, minerals, protein, and calories.
- The final product (trail mix) must have a certain minimum nutritional profile.
- Minimize production cost while satisfying the nutritional profile.

Component	Grams per Pound					Nutritional Requirements
	Seeds	Raisins	Flakes	Pecans	Walnuts	
Vitamins	10	20	10	30	20	20
Minerals	5	7	4	9	2	10
Proteins	1	4	10	2	1	15
Calories	500	450	160	300	500	600
Cost	\$4	\$5	\$3	\$7	\$6	

Covering Models



1) What factors are under our control?

- The package trail mix – amounts of Seeds (S), Raisins (R), Flakes (F), Pecans (P), Walnuts (W)

2) What measure are we trying to optimize?

- Minimize cost of a package.

3) What restrictions limit our choice of decision variables?

- Nutritional Profile
 - Vitamins
 - Minerals
 - Protein
 - Calories



Covering Models

- Decision variables – pound of Seeds (S), Raisins (R), Flakes (F), Pecans (P), Walnuts (W)
- Objective Function – Cost minimization
$$\text{Min } 4S + 5R + 3F + 7P + 6W$$
- Constraints – Subject to
 - Vitamins
$$10S + 20R + 10F + 30P + 20W \geq 20$$
 - Minerals
$$5S + 7R + 4F + 9P + 2W \geq 10$$
 - Protein
$$1S + 4R + 10F + 2P + 1W \geq 15$$
 - Calories
$$500S + 450R + 160F + 300P + 500W \geq 600$$
 - Non-negativity
$$S, R, F, P, \text{ and } W \geq 0$$

Blending Models



- Representing ratios/proportions in Veerman Furniture – Page 229 – Powell & Baker.
- Having 0 chairs for sale is unacceptable to Marketing – something was missing from our original formulation.
- Add a policy constraint: Each of the products must make up at least 25 percent of the products available for sale.
- Total number sold: $C + D + T$; Chairs (C), Desks (D), Tables (T).
- 25% of total sales for chairs: $\frac{C}{C+D+T} \geq 0.25$
- This is not a linear constraint and will make the model nonlinear.



Add a Ratio Constraint

- 25% of total sales for chairs: $\frac{C}{C+D+T} \geq 0.25$
- ~~$(C + D + T) \times \frac{C}{C+D+T} \geq 0.25 \times (C + D + T)$~~
- $C \geq 0.25(C + D + T)$
- $C - 0.25C - 0.25D - 0.25T \geq 0$
 - $0.75C - 0.25D - 0.25T \geq 0$
- Same for Desks (D), Tables (T)
 - $-0.25C + 0.75D - 0.25T \geq 0$
 - $-0.25C - 0.25D + 0.75T \geq 0$



Ratio Constraints

- Model situations where the value of one (or more) variable, compared with the value of another (one or more) variable, must satisfy some relationship.

- **Ratio:** $\frac{x_1}{x_2} \geq 2.5$

- This constraint can occur when a certain product mix ratio is desired between two products.

- **Percentage Constraints:** $\frac{x_1}{x_1+x_2+x_3} \leq 0.25$

- Common in blending problems, where recipes may have a certain amount of flexibility.
- Useful in portfolio investment problems to stay within certain asset allocation guidelines (for example, at most 25% of total portfolio should be invested in bonds).

- **Weighted Average:** $\frac{(20x_1+35x_2)}{(x_1+x_2)} \geq 10$



Linearizing Ratio Constraints

- Ratio: $\frac{x_1}{x_2} \geq 2.5$ Non-linear form
- ~~$x_2 \times \frac{x_1}{x_2} \geq 2.5 \times x_2$~~
- $x_1 \geq 2.5x_2$
- $x_1 - 2.5x_2 \geq 0$ } Linear forms
- Percentage Constraints: $\frac{x_1}{x_1+x_2+x_3} \leq 0.25$ Non-linear form
- ~~$(x_1 + x_2 + x_3) \times \frac{x_1}{x_1+x_2+x_3} \leq 0.25 \times (x_1 + x_2 + x_3)$~~
- $x_1 \leq 0.25(x_1 + x_2 + x_3)$
- $x_1 \leq 0.25x_1 + 0.25x_2 + 0.25x_3$
- $x_1 - 0.25x_1 - 0.25x_2 - 0.25x_3 \leq 0$ } Linear forms

Linearizing Ratio Constraints

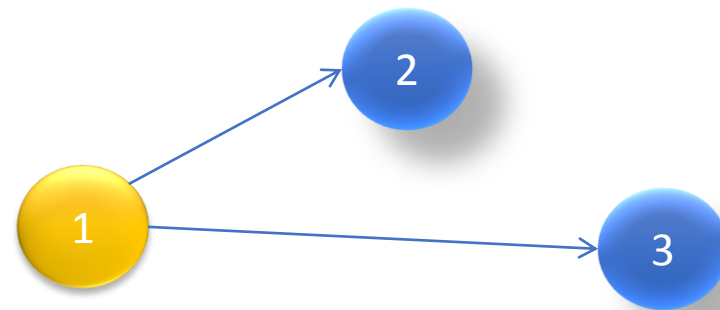


- Weighted Average: $\frac{(20x_1+35x_2)}{(x_1+x_2)} \geq 10$ Non-linear form
- ~~$(x_1+x_2) \times \frac{(20x_1+35x_2)}{(x_1+x_2)} \geq 10 \times (x_1+x_2)$~~
- $20x_1 + 35x_2 \geq 10(x_1 + x_2)$ ← Linear forms
- $20x_1 + 35x_2 \geq 10x_1 + 10x_2$
- $20x_1 + 35x_2 - 10x_1 - 10x_2 \geq 0$
- $10x_1 + 25x_2 \geq 0$

Network Models



- Many business situations involve a ‘flow’ between connected elements.
- Flow might involve materials, funds, information, or just correspondence.
- Elements can be places or times such as cities, months, or production stages.
- Elements can be represented by nodes (circles).
- Direction of flow can be represented by arcs, or arrows.



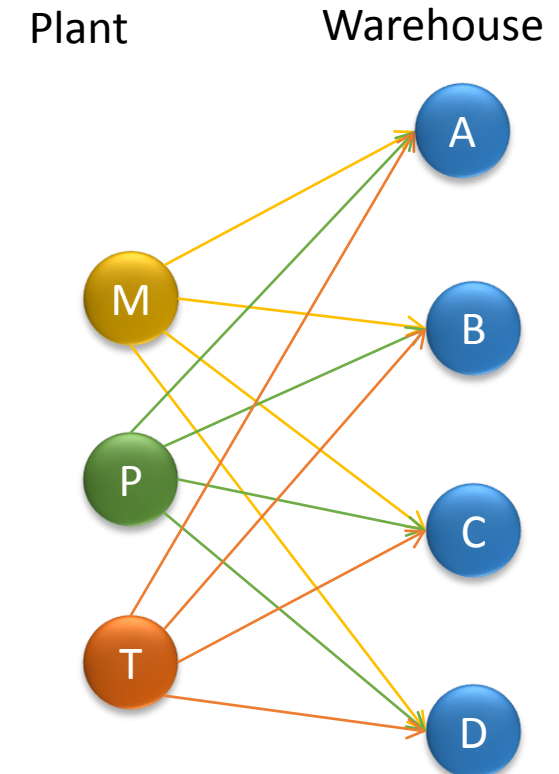
A Network Diagram

Transportation Model



- Transportation model includes:
 - Supply locations (known capacities)
 - Demand locations (known requirements)
 - Unit transportation cost between supply-demand pairs.
- Bonner Electronics – Page 257 – Powell & Baker.
- Planning next week shipment to satisfy supply and demand constraints at minimum cost.

Plant	Warehouse				Capacity
	Atlanta	Boston	Chicago	Denver	
Minneapolis	\$0.60	\$0.56	\$0.22	\$0.40	9,000
Pittsburgh	\$0.36	\$0.30	\$0.28	\$0.58	12,000
Tucson	\$0.65	\$0.68	\$0.55	\$0.42	13,000
Requirement	7,500	8,500	9,500	8,000	





Transportation Model

- Decision variables are number of cartons along each route.
- Objective function – Minimize the total cost.
- Constraints –
 - Minneapolis capacity
 - Pittsburgh capacity
 - Tucson capacity
 - Atlanta demand
 - Boston demand
 - Chicago demand
 - Denver demand
 - Non-negativity

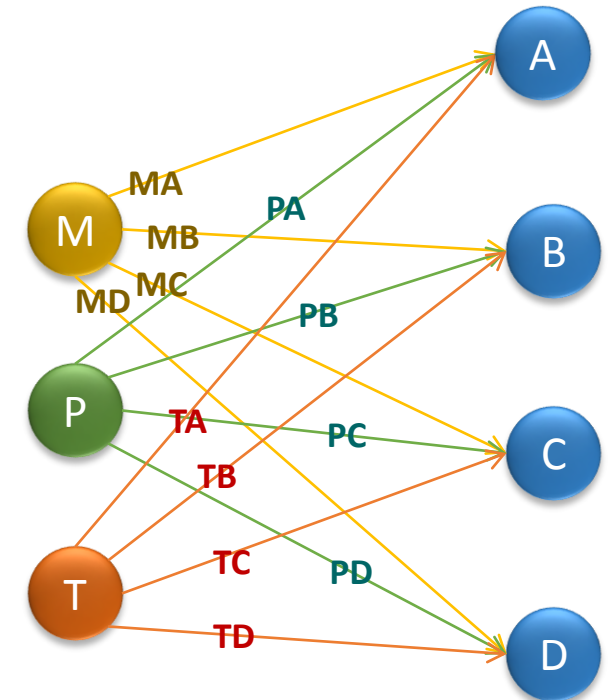
$$\begin{aligned} \text{Min } & 0.60MA + 0.56MB + 0.22MC + 0.40MD \\ & + 0.36PA + 0.30PB + 0.28PC + 0.58PD \\ & + 0.65TA + 0.68TB + 0.55TC + 0.42TD \end{aligned}$$

$$\begin{aligned} MA + MB + MC + MD &\leq 9000 \\ PA + PB + PC + PD &\leq 12000 \\ TA + TB + TC + TD &\leq 13000 \end{aligned}$$

$$\begin{aligned} MA + PA + TA &\geq 7500 \\ MB + PB + TB &\geq 8500 \\ MC + PC + TC &\geq 9500 \\ MD + PD + TD &\geq 8000 \end{aligned}$$

$$MA, MB, MC, MD, PA, PB, PC, PD, TA, TB, TC, \text{ and } TD \geq 0$$

Plant Warehouse





Transshipment Model

- Western Paper company – Page 269 – Powell & Baker.
- Manufacturing paper at three factories (F1, F2, &F3) with known monthly production capacity.
- Products are shipped by rail to depots (D1&D2).
- Repacked products are sent by truck to warehouses (W1,W2,&W3) with known demand.
- Determine the scheduling the material flow at minimum cost.

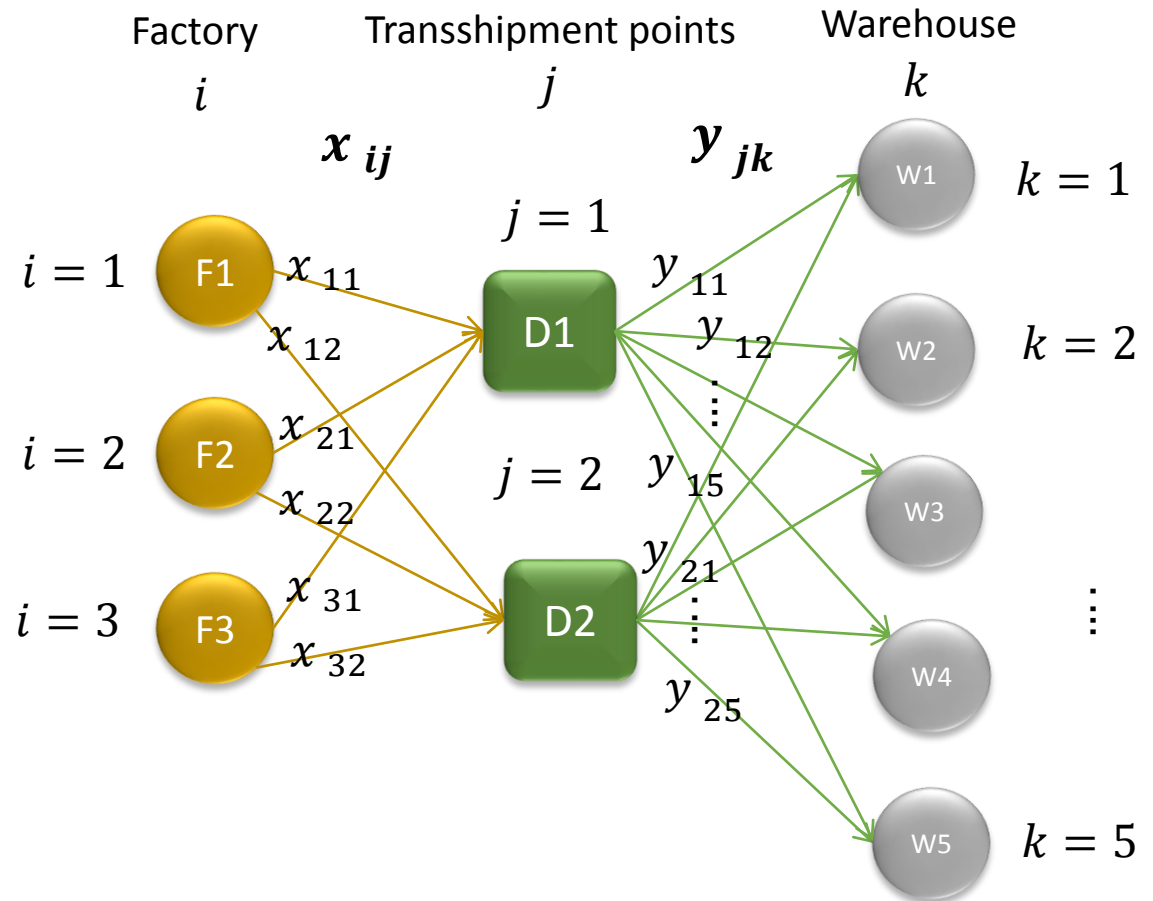
Factory	Depots		Capacity
	D1	D2	
F1	\$1.28	\$1.36	2,500
F2	\$1.33	\$1.38	2,500
F3	\$1.68	\$1.55	2,500

Depots	Warehouse				
	W1	W2	W3	W4	W5
D1	0.60	0.42	0.32	0.44	9,000
D2	0.57	0.30	0.40	0.38	12,000
Requirement	1,200	1,300	1,400	1,500	1,600



Transshipment Model

- Decision variables –
 - First stage x_{ij} : quantity shipped from factories to depots
 - Second stage y_{jk} : quantity shipped from depots to warehouses
- Objective – Minimize total cost
- Constraints –
 - Balance
 - Supply
 - Demand
 - Non-negativity



Balance constraints



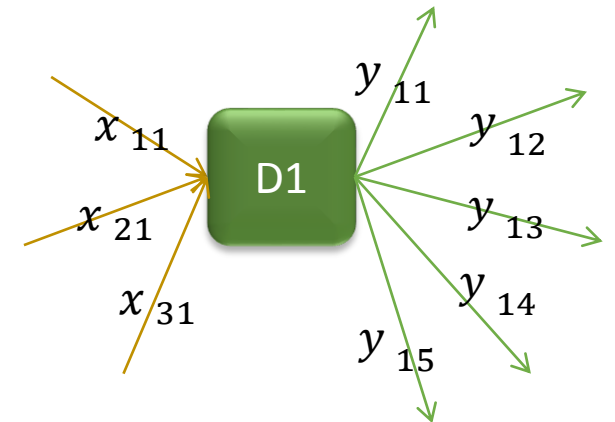
- Used to model processes where the "inputs" must equal the "outputs."
- They have an equality form “=.”
- Also used to model balance between time periods.
- The process of carrying inventory is modeled with a balance constraint.
- Western Paper’s depots are temporary holding points.

Flow Out = Flow In

Flow Out – Flow In = 0

Depot 1: $y_{11} + y_{12} + y_{13} + y_{14} + y_{15} = x_{11} + x_{21} + x_{31}$

$y_{11} + y_{12} + y_{13} + y_{14} + y_{15} - x_{11} - x_{21} - x_{31} = 0$





Transshipment Model

• Objective –

$$\begin{aligned} \text{Min } & 1.28x_{11} + 1.36x_{12} + 1.33x_{21} + 1.38x_{22} + 1.68x_{31} + 1.55x_{32} \\ & + 0.60y_{11} + 0.42y_{12} + 0.32y_{13} + 0.44y_{14} + 0.68y_{15} \\ & + 0.57y_{21} + 0.30y_{22} + 0.40y_{23} + 0.38y_{24} + 0.72y_{25} \end{aligned}$$

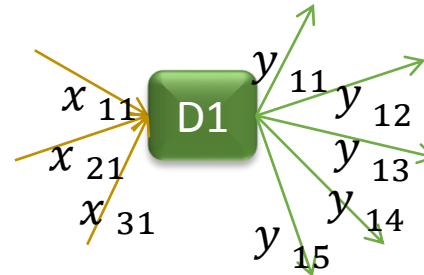
• Constraints –

Supply: at each factory

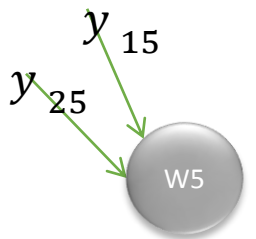
$$\begin{cases} x_{11} + x_{12} \leq 2,500 \\ x_{21} + x_{22} \leq 2,500 \\ x_{31} + x_{32} \leq 2,500 \end{cases}$$

Balance: at each depot

$$\begin{aligned} y_{11} + y_{12} + y_{13} + y_{14} + y_{15} - x_{11} - x_{21} - x_{31} &= 0 \\ y_{21} + y_{22} + y_{23} + y_{24} + y_{25} - x_{12} - x_{22} - x_{32} &= 0 \end{aligned}$$



Demand: at each warehouse



$$\begin{cases} y_{11} + y_{21} \geq 1,200 \\ y_{12} + y_{22} \geq 1,300 \\ y_{13} + y_{23} \geq 1,400 \\ y_{14} + y_{24} \geq 1,500 \\ y_{15} + y_{25} \geq 1,600 \end{cases}$$

Network Models for Process Industries



- Delta Oil company – Page 281 – Powell & Baker.
- Refining process separates crude oil into components that eventually yield gasoline, heating oil, lubricating oil, other petroleum products.
- Distillation tower uses 5 barrels of crude oil to produce 3 barrels of distillate and 2 barrels of low-end by products.
- Some distillate is blended into gasoline products. Rest is feedstock form the catalytic cracker.
- Catalytic cracker produces catalytic gasoline.
- Distillate is blended with catalytic to make regular gasoline and premium gasoline.

Network Models for Process Industries



- Variables:
 - CR: amount of catalytic that is combined into regular gasoline.
 - Crude, Dist, Low, Blend, Feed, Cat, High, BR, BP, CR, CP, Reg, and Prem.

- Balance Equation at T:

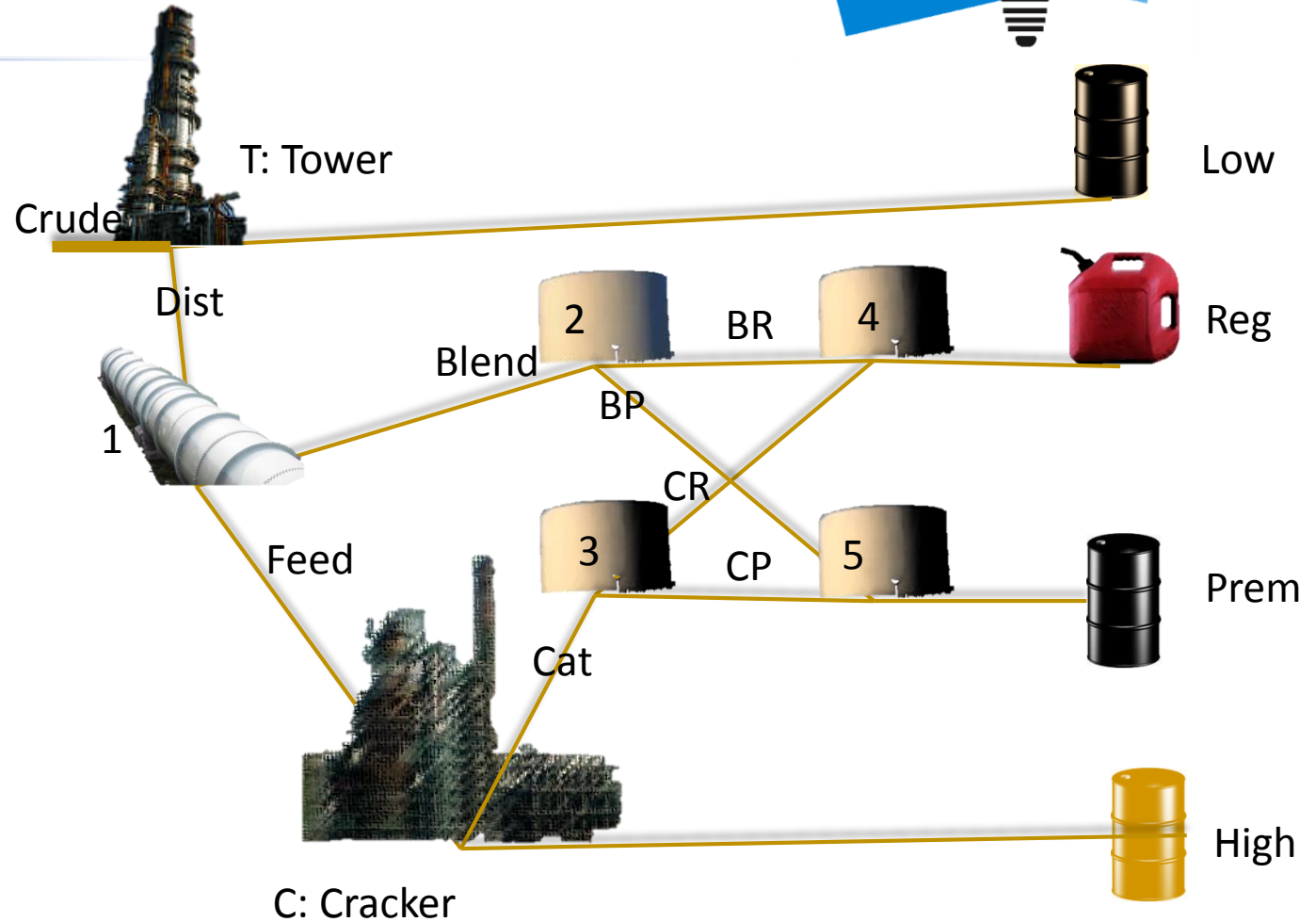
$$Dist - 0.60 Crude = 0$$

$$Low - 0.40 Crude = 0$$
- Balance Equation at C:

$$Cat - 0.64 Feed = 0$$

$$High - 0.40 Feed = 0$$
- Balance Equation at 1:

$$Feed + Blend - Dist = 0$$



Network Models for Process Industries



- Balance Equation at 2,3:

$$BP + BR - Blend = 0$$

$$CP + CR - Cat = 0$$

- Balance Equation at 4,5:

$$Prem - BR - CP = 0$$

$$Reg - BR - CR = 0$$

- Tower Capacity: 50,000
- Cracker Capacity: 20,000
- Sale potential Reg and Prem: 16,000
- Blending floor: Reg 50% catalytic, Prem 70%

- Objective Function:

- Crude oil: \$28 per barrel
- Cost of operating the tower: \$5 per barrel
- Cost of operating the cracker: \$6 per barrel
- Revenue for high-end and low-end byproducts: \$44 and \$36 per barrel
- Revenue for regular and premium gasoline: \$50 and \$55 per barrel

Building Integer Programming Models

Binary Variables and Logical Relationships



Uses of Integer Variables



- Discrete quantities: number of airplanes, cars, houses, or people.
- Yes/No decisions: zero-one (0-1) (or binary) variables. $\begin{cases} \delta = 1 \\ \delta = 0 \end{cases}$
 - $\delta = 1$ indicates that a depot should be build.
 - $\delta = 0$ indicates that a depot should not be build.
- Indicator variables - to impose extra conditions:
 - Use a variable to distinguish between the state where $x = 0$ and $x > 0$.
 - Extra constraints enforce the conditions.
 - Threshold levels.



Capital Budgeting Problem

- Allocating a capital budget, invested in multi-year projects.
- Maximize the value of projects selected, subject to the budget constraint.
- Marr Corporation – Page 292 – Powell & Baker.
- Each project has a required expenditure and a value (NPV of its cash flows over the project life).

	Projects				
	P1	P2	P3	P4	P5
NPV	10	17	16	8	14
Expenditure	48	96	80	32	64



Capital Budgeting Problem

- Decision variables: $y_j = 1$ if project j is accepted; 0 otherwise.
- Objective function: *Maximize* $10y_1 + 17y_2 + 16y_3 + 8y_4 + 14y_5$
- Constraints: subject to:
 - Budget limit $48y_1 + 96y_2 + 80y_3 + 32y_4 + 64y_5 \leq 160$
 - Integer constraint y_j binary

	Projects				
	P1	P2	P3	P4	P5
NPV	10	17	16	8	14
Expenditure	48	96	80	32	64



Relationship Among Variables

- Projects can be related in number of ways:
 - At least m projects must be selected.
 - At most n projects must be selected.
 - Exactly k projects must be selected.
 - Some projects are mutually exclusive.
 - Some projects have contingency relationships.
- Capital budgeting policy constraint:
 - Need to select at least one international project.
 - Projects P2 and P5 are international and others are domestic.
 - Add $y_2 + y_5 \geq 1$ which ensures that combination $y_2 = y_5 = 0$ is not allowed.



Relationship Among Variables

Capital budgeting policy constraints:

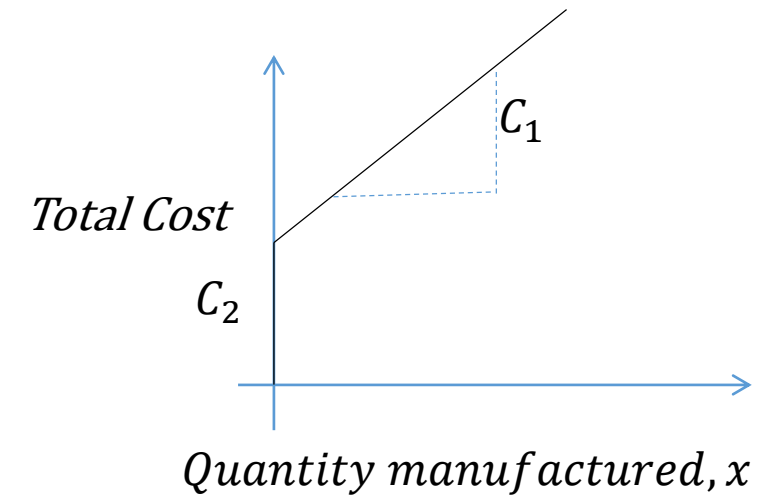
- P4 and P5 are mutually exclusive (they could require same staff resources). $y_4 + y_5 \leq 1$
which ensures that combination $y_4 = y_5 = 1$ is not allowed.
 - Special case of “at most n out of m ”. Here it is 1 out of 2.
- P5 is contingent on P3: P5 requires that P3 be selected. $y_3 - y_5 \geq 0$

P5	P3	Consistent?
0	0	Yes
1	0	Yes
0	1	No
1	1	Yes

Fixed Charge Example



- Example: machine has a start up cost if used at all.
- x represents the quantity of a product to be manufactured at a marginal cost C_1 per unit. If product is manufactured at all, there is a set up cost C_2 to prepare the machine.
 - $x = 0$ total cost = 0
 - $x > 0$ total cost = $C_1x + C_2$
- Total cost is not a linear function of x . It is a discontinuous function.
- Introduce indicator (binary) variable y .
- If any of product manufactured $y=1$.
 - Add $x - My \leq 0$
 - Total cost: $C_1x + C_2y$





Fixed Charge Example

- Allocating capacity, to produce a mix of products.
- Mayhugh Manufacturing – Page 299 – Powell & Baker.
- Maximize the production profit (there is a variable profit and a fixed cost for each product family) subject to demand and production constraints.

	Projects			Hours Available
	F1	F2	F3	
Profit per unit	\$1.20	\$1.80	\$2.20	
Hours Required Per Thousand Units				
Department A	3	4	8	2,000
Department B	3	5	6	2,000
Department C	2	3	9	2,000
Sales Cost (\$000)	60	200	100	
Demand (000)	400	300	50	

Fixed Charge Example



- Decision variables x_1, x_2, x_3 , binary variables y_1, y_2, y_3
- Objective function – Maximize the total profit.
- Constraints –
 - Department A capacity
 - Department B capacity
 - Department C capacity
- Linking constraints
- Integer constraint

$$\text{Max } 1.2 x_1 - 60y_1 + 1.80x_2 - 200y_2 + 2.20 x_3 - 100y_3$$

$$3 x_1 + 4 x_2 + 8 x_3 \leq 2000$$

$$3 x_1 + 5 x_2 + 6 x_3 \leq 2000$$

$$2 x_1 + 3 x_2 + 9 x_3 \leq 2000$$

$$x_1 - My_1 \leq 0$$

$$x_2 - My_2 \leq 0$$

$$x_3 - My_3 \leq 0$$



$$x_1 - 400y_1 \leq 0$$

$$x_2 - 300y_2 \leq 0$$

$$x_3 - 50y_3 \leq 0$$

y_j binary

Threshold Levels



- A common requirement: a decision variable x is either 0, or \geq a specified minimum.
 - This is sometimes called a semi-continuous variable.
- Introduce a 0-1 indicator variable y .
- m is the minimum feasible value of x if it is nonzero. M is an upper limit for x .

$$x - My \leq 0$$

$$x - my \geq 0$$

Therefore:

$$y = 1 \rightarrow m \leq x \leq M$$

$$y = 0 \rightarrow x = 0$$

- Mayhugh Manufacturing – Page 303 – Powell & Baker. $250 \leq x_2 \leq 300$
 $x_2 - 250y_2 \geq 0$

Non-Smooth Models: Hardest to Solve



Linear: =SUM(A1:A10)

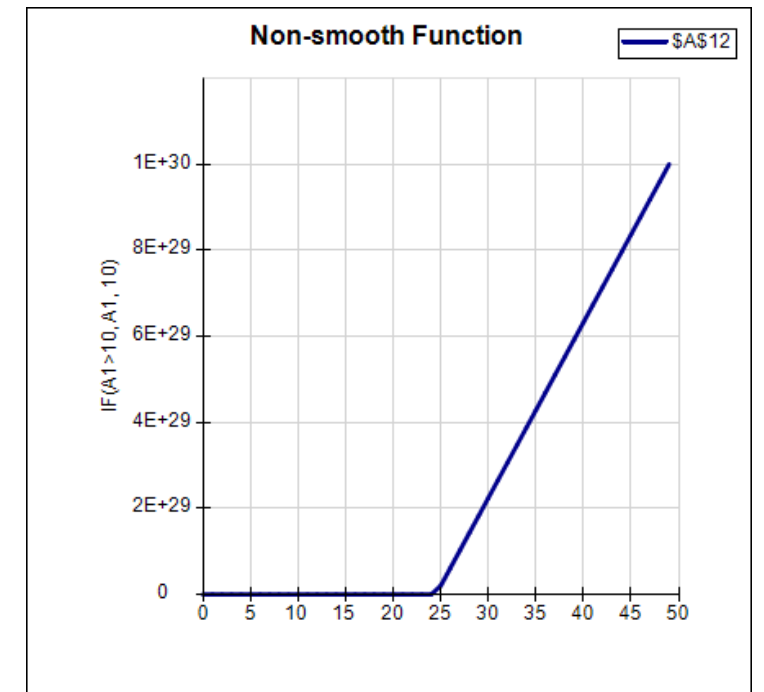
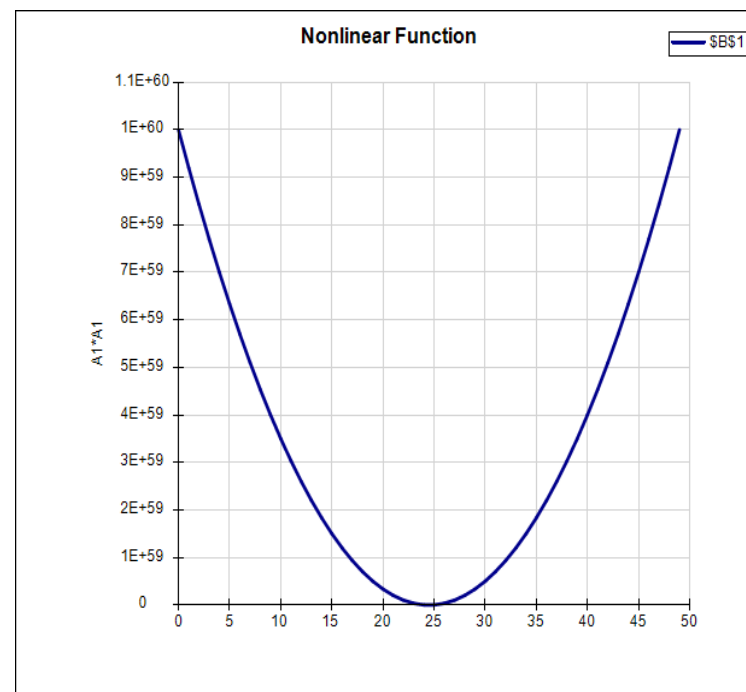
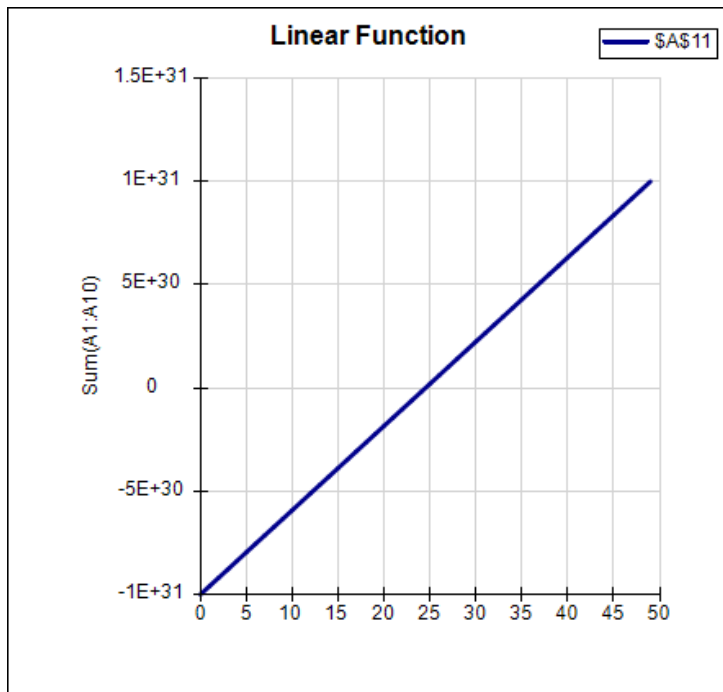
Easiest/Fastest

Non-linear: =A1*A1

Slower

Non-smooth: =IF(A1>25, A1, 0)

Hardest/Slowest



Non-Smooth Transformation



- Applies to the following functions:
 - IF, nested IF, AND, OR, NOT
 - ABS, MIN, MAX
 - CHOOSE, LOOKUP (up to 100 levels)
- Will be used only if result is a linear function.

IF Functions in Spreadsheet



- $=IF(J5 > 500, L5, M5)$, where only J5 depends on the decision variables.
- Introducing a binary integer variable (say H5) that is 1 if the conditional argument of the IF is TRUE, and 0 otherwise.
- Add the constraints:
 - $J5 - 500 \leq M * H5$
 - $500 - J5 \leq M * (1 - H5)$
- Replace IF function with $L5 * H5 + M5 * (1 - H5)$.



IF Functions in Spreadsheet

- $\text{IF}(f(x) > 0, h(x), g(x))$
- We introduce 2 new variables: a binary Y , indicating if $f(x) > 0$, and a new variable r , which will replace the if function completely.
- Then add the constraints:
 - $-M * Y + h(x) \leq r \leq h(x) + M * Y$
 - $-M * (1 - Y) + g(x) \leq r \leq g(x) + M * (1 - Y)$
- To make sure Y has the right value, add:
 - $M * Y + f(x) \geq 0$ (this forces $Y = 1$ when $f(x) < 0$, and is always true otherwise)
 - $-M * (1 - Y) + f(x) \leq 0$ (this forces $Y = 0$ when $f(x) \geq 0$, and is always true otherwise)



Absolute Value (ABS)

- $A1$ is a product tolerance and $C1$ is the desired value.
- $ABS(A1 - C1)$ is the deviation from the desired value.
- **Objective function:** $Min\ ABS(A1 - C1) + \dots$
- Introduce $X = A1 - C1$. So we want: $Min\ ABS(X)$
- Add a new variable Y ; Add constraints:
$$X \leq Y$$
$$-X \leq Y$$
- We replace in objective $Min\ Y$.

MAX()/MIN()



- **Minimize MAX(x_1, x_2)**
- Introduce a new variable Z equivalent to $Max\{x_1, x_2\}$
- Add constraints:
 - $Z \geq x_1, Z \geq x_2$
- We replace in objective Minimize Z
- **Maximize MIN(x_1, x_2)**
- Introduce a new variable Z equivalent to $Min\{x_1, x_2\}$
- Add constraints:
 - $Z \leq x_1, Z \leq x_2$
- We replace in objective Maximize Z

Benefits of Optimization



- A properly formulated model will give you the benefits of an optimal solution.
- Most common business situations can be modeled using the techniques we discussed today.
- Ideally, start the modeling process with these techniques in mind.
- For existing models, Frontline's Solvers can automatically transform IF, CHOOSE, LOOKUP, and other functions into linear integer form.

Further Benefits of Modeling



- Building a model often reveals relationships and yields a greater understanding of the situation being modeled.
- Having built a model, it is possible to apply analytic methods to suggest courses of action that might not otherwise be apparent.
- Experimentation is possible with a model, whereas it is often not possible, or desirable, to experiment with the situation being modeled.
- Analytic Solver Platform is a complete toolset for descriptive, predictive and prescriptive analytics.

Contact Info



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- You may also download this presentation from our website at www.solver.com/training/premsolver-2.
- You can download a free trial version of Analytic Solver Platform at Solver.com.

References



- **Management Science-The Art of Modeling with Spreadsheets, 4th Edition**
<http://www.wiley.com/WileyCDA/WileyTitle/productCd-EHEP002883.html>
- **Spreadsheet Modeling and Decision Analysis: A Practical Introduction to Business Analytics, 7th Edition**
<http://www.cengage.com/us/>
- **Essentials of Business Analytics, 1st Edition**
<http://www.cengage.com/us/>
- **Model Building in Mathematical Programming**
<http://www.wiley.com/WileyCDA/WileyTitle/productCd-1118443330.html>
- **Absolute Value Cases**
<http://lpsolve.sourceforge.net/5.1/absolute.htm>





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Q & A



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Thank You!